

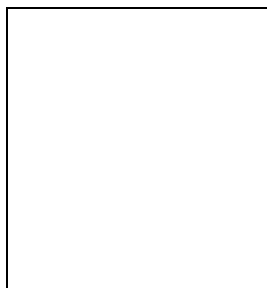
# DIS AT LOW $x$ , SATURATION SCALE, GLUON STRUCTURE FUNCTION AND VECTOR-MESON PRODUCTION <sup>a</sup>

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Deep inelastic scattering at low  $x$  can be described by essentially only two fitted parameters. The interpretation of  $J/\psi$  photoproduction in terms of the gluon structure function is elaborated upon.

I will concentrate on the intimate connection between the  $x$ -dependence and the  $Q^2$  dependence of the structure function  $F_2(x, Q^2)$ , and subsequently I will turn to vector-meson production, to  $J/\psi$  production in particular.

In deep inelastic scattering (DIS) at low  $x \simeq Q^2/W^2 \ll 0.1$ , the photon fluctuates into a  $q\bar{q}$  color-dipole state that in the virtual forward-Compton-scattering amplitude interacts <sup>1</sup> via the generic structure of two-gluon exchange with the proton. The QCD gauge-theory structure implies diagonal and off-diagonal transitions <sup>2,3</sup> in the masses of the color-dipole vector states, and accordingly it implies a dependence on the transverse three-momentum of the gluon,  $\vec{l}_\perp$ , that couples to the color dipole. The effective value of  $\vec{l}_\perp$  introduces a novel scale, the saturation scale, relevant in low- $x$  DIS. In our approach, the saturation scale,  $\Lambda_{sat}^2(W^2)$ , depends on the energy,  $W$ , and <sup>4</sup>

$$\Lambda_{sat}^2(W^2) = \frac{1}{6} \langle \vec{l}_\perp^2 \rangle \cong \frac{1}{6} \text{const} \left( \frac{W^2}{1 \text{GeV}^2} \right)^{C_2}. \quad (1)$$

A fit to the total photoabsorption cross section by the power law (1) in the HERA energy range gave <sup>3</sup>

$$2 \text{GeV}^2 \lesssim \Lambda_{sat}^2(W^2) \lesssim 7 \text{GeV}^2, \quad (2)$$

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where

$$\begin{aligned} const &= 0.340 \pm 0.063 GeV^2, \\ C_2 &\equiv C_2^{exp} = 0.27 \pm 0.01. \end{aligned} \quad (3)$$

In addition to  $\Lambda_{sat}^2(W^2)$ , the total (virtual) photoabsorption cross section depends on the cross section  $\sigma^{(\infty)}$  of hadronic size,

$$\begin{aligned} \sigma^{(\infty)} &= 48 GeV^{-2} = 18.7 mb, \\ (\text{for } R_{e^+e^-} &= 10/3, \text{ four flavours}), \end{aligned} \quad (4)$$

and is approximately given by

$$\sigma_{\gamma^*p}(W^2, Q^2) \cong \frac{\alpha}{3\pi} R_{e^+e^-} \sigma^{(\infty)} \cdot \begin{cases} \ln \eta^{-1}, & (\eta \ll 1), \\ \frac{1}{2} \eta^{-1}, & (\eta \gtrsim 1), \end{cases} \quad (5)$$

with the scaling variable<sup>5,3</sup>

$$\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)} \quad (6)$$

and  $m_0^2 \simeq 0.15 GeV^2$ . Apart from this threshold mass, the cross section (5), or equivalently  $F_2(W^2, Q^2)$ , contains three adjusted parameters, the two parameters (3) determining the saturation scale and the cross section (4).

Application of DGLAP evolution in the region of  $Q^2 \gg \Lambda_{sat}^2(W^2)$ , where appropriate, actually reduces the number of three to only two adjusted parameters, since evolution allows one to determine the exponent  $C_2$  in (1). This will be pointed out next.

The representation (5) of the experimental data contains the assumption that the scattering amplitude for longitudinal,  $(q\bar{q})_L^{J=1}$ , (vector) states and for transverse ones,  $(q\bar{q})_T^{J=1}$ , be proportional to each other. In terms of the sea-quark,  $x\Sigma(x, Q^2)$ , and the gluon distribution,  $xg(x, Q^2)$ , and the proportionality constant  $r$ , this proportionality reads<sup>6</sup>

$$\begin{aligned} x\Sigma(x, Q^2) &= \frac{12}{R_{e^+e^-}} F_2(x, Q^2) = \frac{8}{3\pi} \left( r + \frac{1}{2} \right) \alpha_s(Q^2) xg(x, Q^2)|_{x=Q^2/W^2} \\ &= \frac{1}{3\pi^3} \left( r + \frac{1}{2} \right) \sigma^{(\infty)} \Lambda_{sat}^2(W^2). \end{aligned} \quad (7)$$

The constant  $r$  also determines the ratio of the longitudinal to the transverse photoabsorption cross section,

$$\frac{\sigma_{\gamma_L^*p}(W^2, Q^2)}{\sigma_{\gamma_T^*p}(W^2, Q^2)} = \frac{1}{2r}. \quad (8)$$

The (successful) representation<sup>3</sup> of the experimental data was based on  $r = 1$ . With (5) and (7), the evolution equation (at low  $x$ )<sup>7</sup>

$$\frac{\partial F_2(\frac{x}{2}, Q^2)}{\partial \ln Q^2} = \frac{R_{e^+e^-}}{9\pi} \alpha_s(Q^2) xg(x, Q^2) \quad (9)$$

turns into an equation for  $\Lambda_{sat}^2(W^2)$ . Inserting the power law (1), one finds a constraint on  $C_2$  that is given by<sup>6</sup>

$$(2r + 1)2^{C_2} C_2 = 1. \quad (10)$$

In Table 1, we show the relation between  $r$  and  $C_2$  resulting from (10). The constant  $r$ , according to (7), determines the relative magnitude of gluon to sea distribution. The dependence of the structure function  $F_2(W^2) = F_2(Q^2/x)$  for  $Q^2 \gg \Lambda_{sat}^2(W^2)$  follows from (5).

Table 1: Results for  $C_2^{theor.}$  for different values of  $r$  according to (10).

r	$C_2^{theor.}$	$\alpha_s \cdot \text{glue}$	$\sigma_{\gamma_L^*}/\sigma_{\gamma_T^*}$	$F_2\left(\frac{Q^2}{x}\right)$
$\rightarrow \infty$	0	$\ll \text{sea}$	0	$(Q^2/x)^0 = \text{const.}$
1	0.276	$\approx \text{sea}$	$\sim \frac{1}{2}$	$(Q^2/x)^{0.276}$
0	0.65	$> \text{sea}$	$\infty$	$(Q^2/x)^{0.65}$

We summarize:

- i) The theoretical value of  $C_2$  in Table 1 from (9) and (10) for  $r = 1$  coincides with the experimental one (3) obtained for  $r = 1$ ,

$$C_2^{theor.} \simeq C_2^{exp.},$$

and thus the underlying ansatz for the dipole cross section is consistent with the evolution equations from QCD. A (strong) violation of (10) would have ruled out this ansatz, and in particular the underlying assumption of  $W$  being the relevant variable to describe diffractive processes at low  $x$ .

- ii) Essentially two parameters, the normalization of the saturation scale  $\Lambda_{sat}^2(W^2)$  in (3) and the cross section of hadronic magnitude (4) are sufficient to determine the low- $x$  proton structure function including the photoproduction limit.
- iii) The  $Q^2$  and the  $x$  dependence of  $F_2(x, Q^2)$  are strongly correlated with each other and correlated with the relative magnitude of the gluon and sea contributions, compare Table 1.
- iv) A sufficiently large gluon contribution implies a strong rise of  $F_2(x, Q^2)$  with increasing  $Q^2$  for constant  $x$ , and an equally strong rise with decreasing  $x$  at fixed  $Q^2$  (compare lines 2 and 3 in Table 1). This qualitative feature is experimentally realized, and theoretically it is a natural consequence of  $W$  as the relevant variable that describes the scattering cross section of a color dipole on the proton (rather than  $x$ ).
- v) Since the relative magnitude of the gluon and the sea is correlated with  $\sigma_{\gamma_L^* p}/\sigma_{\gamma_T^* p}$ , direct measurements of this ratio are urgently needed. This allows one to investigate the limits of validity of the underlying assumed proportionality of sea and gluon distributions.

Turning to  $J/\psi$  production, in figs. 1 and 2, I show our result<sup>8,4</sup> of an absolute prediction based on the description of the inclusive DIS data I told you about. For details, I have to refer to the original publications.

I wish to mention one important point, however, related to the interpretation of  $J/\psi$  photoproduction ( $Q^2 = 0$ ) in terms of the gluon structure function. From (7), valid for sufficiently large  $Q^2 \gg \Lambda_{sat}^2(W^2)$ , we have

$$\alpha_s(Q^2) x g(x, Q^2)|_{x=Q^2/W^2} = \frac{1}{8\pi^2} \sigma^{(\infty)} \Lambda_{sat}^2(W^2 = Q^2/x). \quad (11)$$

According to (11), a determination of the energy dependence of  $\Lambda_{sat}^2(W^2)$  at any  $Q^2$ , e.g. at  $Q^2 = 0$  in  $J/\psi$  photoproduction, yields the dependence of the gluon structure function on the left-hand side as a function of  $x$  at  $Q^2 \gg \Lambda_{sat}^2(W^2)$ , where relation (11) becomes valid. Clearly, the measurement of  $J/\psi$  photoproduction does *not* provide a measurement of the structure function for  $Q^2 \lesssim m_c^2, \Lambda_{sat}^2(W^2)$ , where (11) breaks down.

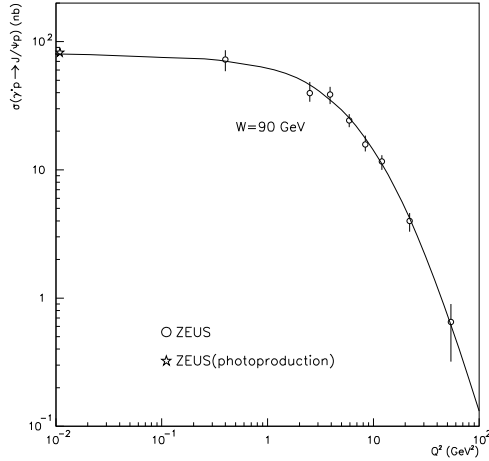


Figure 1: The  $Q^2$  dependence of the cross section for  $J/\psi$  production.

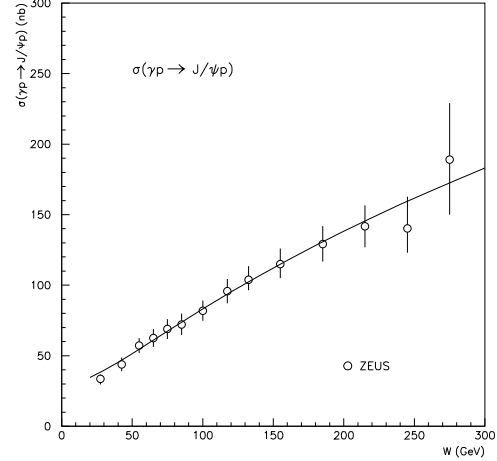


Figure 2: The  $W$ -dependence of  $J/\psi$  photoproduction

More generally, independent of *our* representation of the data on DIS, *any* unique prediction of  $J/\psi$  photoproduction necessarily requires the left-hand side of (11) to only depend on  $W^2$ . Otherwise no unique prediction of  $J/\psi$  photoproduction will emerge. This should be kept in mind, when predicting the energy dependence of vector meson photoabsorption, i.e. for *any* specific fit of the gluon structure function the left-hand side in (11) should be examined on whether it only depends on  $W^2$  in good approximation at large  $Q^2$ .

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